A General Scattering Theory

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Abstract—A fundamentally different approach to the problems of electromagnetic wave propagation through a medium containing a distribution of scatterers is presented. The new theory includes the effects of scatterers in the near field of the transmitting and/or receiving antennas, a factor that currently accepted theories do not include. Near-field effects are important especially in the case of satellite microwave communications where the near field of the large ground antennas includes an appreciable fraction or even the whole of the rain path. The theory is based on the Lorentz form of Reciprocity Theorem and may be used to estimate attenuation and depolarization of microwaves through precipitation.

1. Introduction

Current interest in microwave propagation studies through precipitation particles has been prompted by proposals for terrestrial and satellite communication systems operating above 10 GHz.[1,2] At these frequencies the presence of precipitation particles in the transmission medium causes attenuation and depolarization of the transmitted radiation.[3,4] Both effects may represent a severe limitation on system performance, and in particular, the depolarization effect is of considerable importance to the possible use of two orthogonal polarizations as separate communication channels in future satellite and terrestrial communication systems.[5]

A theoretical approach to the problem of microwave attenuation due to various meteorological phenomena was first given by Ryde[6] in 1946. In his paper Ryde computed the attenuation and the intensity of the radar echoes produced by fog, cloud, rain, hail, and

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snow on the basis of electromagnetic theory for wavelengths in the centimeter band. He showed that the principal cause of microwave attenuation is rain. Ryde assumed spherical drops throughout his work and, therefore, depolarization effects due to departure of raindrop shape from sphericity were not reported.

Van de Hulst[7] treated a similar problem of wave propagation in a medium containing independent particles that scatter and absorb the incident energy. The effect of the particles on the transmitted wave is expressed in terms of an effective complex index of refraction. Van de Hulst's approach is very useful in that it predicts both the attenuation and phase shift introduced by the particles. Both are important in estimating rain-induced depolarization of the transmitted radiation.

In 1965 Medhurst[8] repeated and extended Ryde's work on rain. He made a systematic comparison between theoretically predicted rainfall attenuation levels and those found by experiment. He concluded that the agreement was not completely satisfactory, and that there was a marked tendency for observed attenuations to occur well above levels which according to the theoretical predictions could not be exceeded. Medhurst gave two possible reasons for this. The first reason was the neglect in the theory of multiple scattering effects along the path. Ryde had assumed that the interaction between drops was negligible when the distance between the drops was greater than five times their diameter, as normally will be the case. The second possible source of error in the theory was that the rain structure was more complex than had been assumed, in that the precipitation may tend to contain clusters of two or more closely spaced drops.

Mink[9], using a controlled experiment, showed however that for no variations in the rainfall rate and drop-size composition along the transmission path, there still existed a large discrepancy between theory and measurements.

This observation was also taken up by Crane,[10] who suggested that Van de Hulst's theory was incomplete in that it did not include the effects that rain has on microwave radiation when in the near field of an antenna. Crane modified the existing theory by including antenna pattern correction factors for the near field. He further supported his theoretical modifications with experimental evidence. He concluded that rain in the near field of large antenna systems may cause different values of attenuation than predicted on the basis of Van de Hulst's theory. His results also showed measurable polarization effects both for phase and amplitude.

In 1978 Haworth et al[11] expressed doubt as to the correctness of

Crane's conclusions. They suggested that the antenna correction factors of Crane resulted from the neglect of the near-field effects on the boresight amplitudes and especially phases of the antenna responses, and that when proper account is taken of these effects the antenna correction factors will disappear. They commented on the importance of conclusively clarifying the effects of rain in the near field of an antenna especially for microwave satellite communications.

Knowledge of whether rain in the near field of an antenna has different effects on microwave propagation than in the far field is of extreme importance. Although in terrestrial microwave communications the near field of both antennas might be only a small portion of the total path of the link, in satellite microwave communications the near field of the large ground-station antenna could include an appreciable fraction or even the whole of the rain path.

In this paper, we investigate the scattering effects that rain, or any other scatterer, in the near field of one or both antennas has on a microwave communication link. From this investigation a new theoretical approach to the problem of electromagnetic wave propagation through a medium containing a distribution of scatterers has been developed. The new theory includes this effect of the near field regions of the two antennas; it is found that their contribution is negligible.

2. Analysis

The analysis is given in four sections. Sec. 2.1 deals with propagation in the absence of precipitation and an expression for the received wave is derived. In Sec. 2.2, a raindrop is introduced and the received wave due to its scattered radiation is deduced. Sec. 2.3 considers the received wave due to a uniform distribution of similar sized raindrops contained within an elemental volume. Sec. 2.4 extends the analysis to obtain expressions for the attenuation and phase shift due to a thick precipitation layer.

2.1 E. M. Wave Propagation in the Absence of Precipitation

Consider the situation of two aperture antennas directed towards each other as shown in Fig. 1. Let $\mathbf{E}_1,\mathbf{H}_1$ be the field of the transmitter (antenna 1) and $\mathbf{E}_2,\mathbf{H}_2$ be the field of the receiver (antenna 2) when it acts as a transmitter.

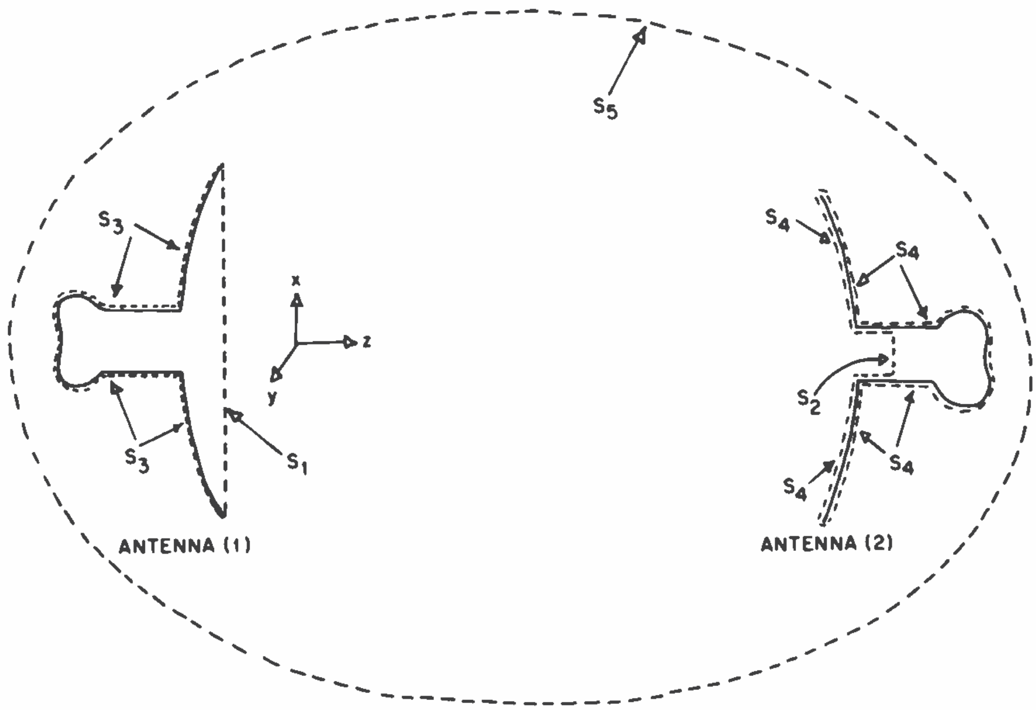

Fig. 1—Propagation in the absence of precipitation.

Consider the volume V, bounded by the surface S_1, S_2, S_3, S_4, and S_5. Within this volume there are no impressed currents and both fields ($\mathbf{E}_1$,$\mathbf{H}_1$ and $\mathbf{E}_2$,$\mathbf{H}_2$) satisfy Maxwell's equations in free space i.e.,

$$\begin{aligned}
\nabla \times \mathbf{E}_1 &= -jw\,\mu_0\,\mathbf{H}_1 \\
\nabla \times \mathbf{H}_1 &= jw\,\epsilon_0\,\mathbf{E}_1 \\
\nabla \times \mathbf{E}_2 &= -jw\,\mu_0\,\mathbf{H}_2 \\
\nabla \times \mathbf{H}_2 &= jw\,\epsilon_0\,\mathbf{E}_2
\end{aligned} \qquad [1]$$

Also

$$\nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) = \mathbf{H}_2 \cdot \nabla \times \mathbf{E}_1 - \mathbf{E}_1 \cdot \nabla \times \mathbf{H}_2 - \mathbf{H}_1 \cdot \nabla \times \mathbf{E}_2 + \mathbf{E}_2 \cdot \nabla \times \mathbf{H}_1 \qquad [2]$$

Using Eq. [2] in Eqs. [1], we obtain

$$\begin{aligned}
\nabla \cdot (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) &= -jw\,\mu_0\,\mathbf{H}_1 \cdot \mathbf{H}_2 - jw\,\epsilon_0\,\mathbf{E}_1 \cdot \mathbf{E}_2 \\
&\quad + jw\,\mu_0\,\mathbf{H}_1 \cdot \mathbf{H}_2 + jw\,\epsilon_0\,\mathbf{E}_1 \cdot \mathbf{E}_2 \\
&= 0.
\end{aligned}$$

Hence, using Gauss' theorem, we have

$$\iint_{S_1+S_2+\ldots S_5} (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot \hat{\mathbf{n}} dS = 0, \qquad [3]$$

where $\hat{\mathbf{n}}$ is taken as the outward normal on the respective surfaces. Equation [3] is in fact the Lorentz form of Reciprocity Theorem for free space.

Consider now each surface in turn.

(a) Surface S_1

The surface S_1 is taken as the aperture plane of antenna 1. Assuming large antennas, compared to the wavelength, and assuming matched polarizations, the fields over this surface can be written as,

$$\mathbf{E}_1 = \hat{\mathbf{i}}\, E_1$$
$$\mathbf{H}_1 = \hat{\mathbf{j}}\, \sqrt{\epsilon_0/\mu_0}\, E_1$$
$$\mathbf{E}_2 = \hat{\mathbf{i}}\, E_2$$
$$\mathbf{H}_2 = -\hat{\mathbf{j}}\sqrt{\epsilon_0/\mu_0}\, E_2.$$

Also, for this surface, $\hat{\mathbf{n}} = -\hat{\mathbf{k}}$. Hence

$$\iint_{S_1} (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot \hat{\mathbf{n}} dS = 2\sqrt{\epsilon_0/\mu_0} \iint_{S_1} \mathbf{E}_1\, E_2\, dS. \qquad [4]$$

(b) Surface S_2

This surface is located in the transmission line connected to antenna 2. It is assumed, for simplicity of analysis, that the line is matched and only the normal transmission line mode exists. Hence, on surface S_2 we have,

$$\mathbf{E}_1 = \hat{\mathbf{u}}\, B_i\, f(x,y)$$
$$\mathbf{H}_1 = (\hat{\mathbf{k}} \times \hat{\mathbf{u}}) \frac{B_i}{Z_0} f(x,y)$$
$$\mathbf{E}_2 = \hat{\mathbf{u}}\, A\, f(x,y)$$
$$\mathbf{H}_2 = -(\hat{\mathbf{k}} \times \hat{\mathbf{u}}) \frac{A}{Z_0} f(x,y) \qquad [5]$$

where B_i is the complex amplitude of inward travelling wave due to antenna 1 transmitting, A is the complex amplitude of outward travelling wave due to antenna 2 transmitting, Z_0 the wave impedance of the transmission line, $\hat{\mathbf{u}}$ a unit vector normal to $\hat{\mathbf{k}}$, $f(x,y)$ the distribution function of the transmission line mode.

Also, $\hat{\mathbf{n}} = \hat{\mathbf{k}}$. Then

$$(\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot \hat{\mathbf{n}} = -2\frac{A\,B_i}{Z_0} f^2(x,y)\,[\hat{\mathbf{u}} \times (\hat{\mathbf{k}} \times \hat{\mathbf{u}})] \cdot \hat{\mathbf{k}}$$
$$= -2\frac{A\,B_i}{Z_0} f^2(x,y)\,[(\hat{\mathbf{k}} \times \hat{\mathbf{u}}) \cdot (\hat{\mathbf{k}} \times \hat{\mathbf{u}})]$$
$$= -2\frac{A\,B_i}{Z_0} f^2(x,y).$$

Hence

$$\iint_{S_2} (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot \hat{\mathbf{n}} dS = -2\,\frac{A\,B_i}{Z_0} \iint_{S_2} f^2(x,y)\,dx\,dy. \quad [6]$$

(c) Surfaces S_3 and S_4

The integral over these surfaces vanishes. To show this, we note that $\mathbf{E}_1$ and $\mathbf{E}_2$ are parallel with the normal $\hat{\mathbf{n}}$ everywhere over S_3 and S_4 which are assumed to be perfectly conducting surfaces. Hence we can write $\mathbf{E}_1 = \hat{\mathbf{n}} E_1$ and $\mathbf{E}_2 = \hat{\mathbf{n}} E_2$ over these surfaces. Thus

$$(\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot \hat{\mathbf{n}} = [E_1\,(\hat{\mathbf{n}} \times \mathbf{H}_2) \cdot \hat{\mathbf{n}} - E_2\,(\hat{\mathbf{n}} \times \mathbf{H}_1) \cdot \hat{\mathbf{n}}]$$
$$= [E_1\,(\hat{\mathbf{n}} \times \hat{\mathbf{n}}) \cdot \mathbf{H}_2 - E_2\,(\hat{\mathbf{n}} \times \hat{\mathbf{n}}) \cdot \mathbf{H}_1]$$
$$= 0.$$

Therefore

$$\iint_{S_3+S_4} (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot \hat{\mathbf{n}} dS = 0. \quad [7]$$

(d) Surface S_5

Surface S_5 is taken to be a large sphere tending to infinity. Hence on this surface we have

$$\mathbf{E}_1 = -\hat{\mathbf{n}} \times \mathbf{H}_1 \sqrt{\mu_0/\epsilon_0}$$
$$\mathbf{E}_2 = -\hat{\mathbf{n}} \times \mathbf{H}_2 \sqrt{\mu_0/\epsilon_0}$$

and

$$(\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot \hat{\mathbf{n}} = -\sqrt{\mu_0/\epsilon_0}\,[(\hat{\mathbf{n}} \times \mathbf{H}_1) \times \mathbf{H}_2] \cdot \hat{\mathbf{n}} + \sqrt{\mu_0/\epsilon_0}\,[(\hat{\mathbf{n}} \times \mathbf{H}_2) \times \mathbf{H}_1] \cdot \hat{\mathbf{n}}$$
$$= \sqrt{\mu_0/\epsilon_0}\,[\hat{\mathbf{n}}\,(\mathbf{H}_2 \cdot \mathbf{H}_1) - \mathbf{H}_1\,(\mathbf{H}_2 \cdot \hat{\mathbf{n}})] \cdot \hat{\mathbf{n}} - \sqrt{\mu_0/\epsilon_0}\,[\hat{\mathbf{n}}\,(\mathbf{H}_1 \cdot \mathbf{H}_2) - \mathbf{H}_2\,(\mathbf{H}_1 \cdot \hat{\mathbf{n}})] \cdot \hat{\mathbf{n}}$$
$$= 0.$$

Therefore

$$\iint_{S_5} (\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) \cdot \hat{\mathbf{n}} dS = 0. \qquad [8]$$

Finally, using Eqs. [4], [6], [7], and [8], we have

$$2\sqrt{\epsilon_0/\mu_0} \iint_{S_1} E_1 E_2 \, dS - 2 \frac{A B_i}{Z_0} \iint_{S_2} f^2(x,y) \, dS = 0,$$

i.e.,

$$B_i = \frac{Z_0}{A} \sqrt{\epsilon_0/\mu_0} \left(\iint_{S_1} E_1 E_2 \, dS \left[\iint_{S_1} f^2(x,y) \, dS \right]^{-1} \right). \qquad [9]$$

2.2 E. M. Wave Propagation In the Presence of a Single Particle

Consider the arrangement shown in Fig. 2. It is similar to Fig. 1 except that an arbitrarily shaped raindrop (or any scatterer) is now introduced.

Let $\mathbf{E}_3,\mathbf{H}_3$ be the field due to the transmitter (antenna 1) in the presence of precipitation and $\mathbf{E}_2,\mathbf{H}_2$ be the field of the receiver (antenna 2) when it acts as a transmitter in free space. (N.B., $\mathbf{E}_2,\mathbf{H}_2$ here are the same as defined in Sec. 2.1; however, $\mathbf{E}_3,\mathbf{H}_3$ are not the same as $\mathbf{E}_1,\mathbf{H}_1$ defined in the same section. This will become clear in Sec. 2.4.) Let $\mathbf{E}_{S1},\mathbf{H}_{S1}$ be the scattered field in free space due to $\mathbf{E}_3,\mathbf{H}_3$ incident on the raindrop. Both $\mathbf{E}_{S1},\mathbf{H}_{S1}$ and $\mathbf{E}_2,\mathbf{H}_2$ satisfy Maxwell's free-space equations in the volume V bounded by the surfaces S_2, S_3 . . . S_7. Hence, following the initial analysis in Sec. 2.1, we have

$$\iint_{S_2+S_3+\ \ldots S_7} (\mathbf{E}_{S_1} \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_{S_1}) \cdot \hat{\mathbf{n}} dS = 0 \qquad [10]$$

where $\hat{\mathbf{n}}$ is the outward normal on the respective surfaces.

Consider now the various surface integrals.

<u>(a) Surfaces S_3, S_4, and S_5</u>

By a similar analysis to that given in Sec. 2.1,

$$\iint_{S_3+S_4+S_5} (\mathbf{E}_{S_1} \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_{S_1}) \cdot \hat{\mathbf{n}} dS = 0. \qquad [11]$$

(b) Surface S_2

Also following Sec. 2.1, it can be shown that

$$\iint_{S_2} (\mathbf{E}_{S_1} \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_{S_1}) \cdot \hat{\mathbf{n}} dS = -2 \frac{A B_s}{Z_0} \iint_{S_2} f^2(x,y)\, dS, \quad [12]$$

where A, Z_0 and $f(x,y)$ are the same as defined previously in Sec. 2.1 and B_s is the complex amplitude of the inward travelling wave at S_2 due to the scattered field $\mathbf{E}_{S_1},\mathbf{H}_{S_1}$ which itself is due to $\mathbf{E}_3,\mathbf{H}_3$.

(c) Surface S_7

Both $\mathbf{E}_{S_1},\mathbf{H}_{S_1}$, and $\mathbf{E}_2,\mathbf{H}_2$ give rise to travelling waves in the same direction across S_7. This fact may be used to show

$$\iint_{S_7} (\mathbf{E}_{S_1} \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_{S_1}) \cdot \hat{\mathbf{n}} dS = 0. \quad [13]$$

(d) Surface S_6

This surface is taken to be that of the raindrop. Consider a point 0 anywhere in the scatterer as shown in Fig. 2. At this point, the fields $\mathbf{E}_3,\mathbf{H}_3$ and $\mathbf{E}_2,\mathbf{H}_2$ can be written as

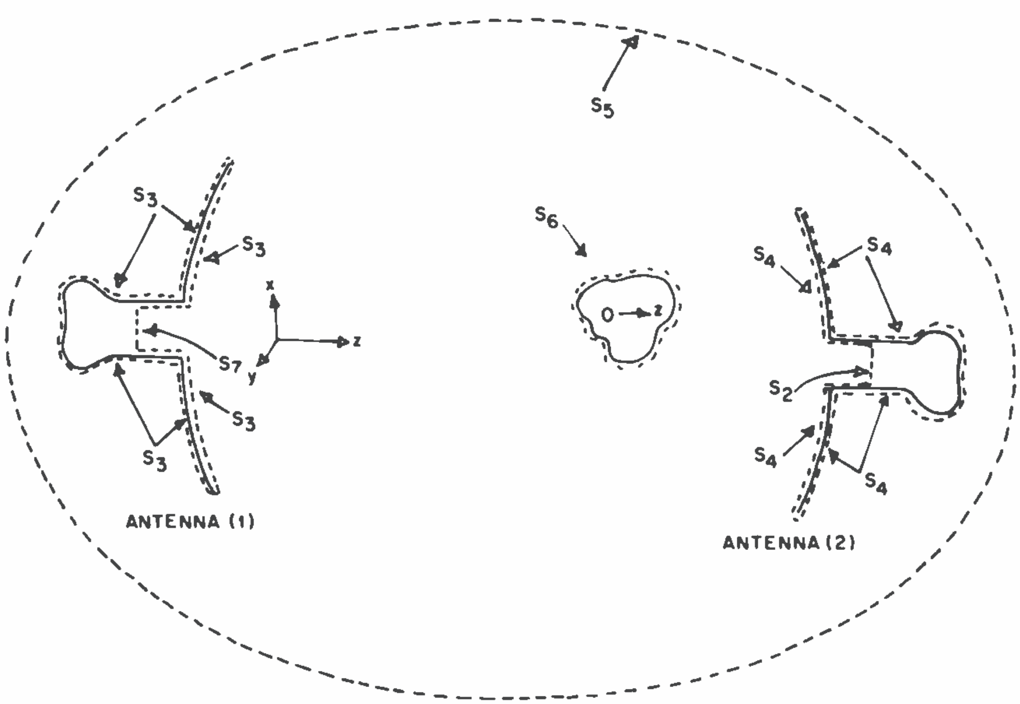

Fig. 2—Propagation in the presence of a single scatterer.

$$\begin{aligned} \mathbf{E}_3 &= \hat{\mathbf{i}}\, E_{30} \\ \mathbf{H}_3 &= \hat{\mathbf{j}}\, E_{30}\sqrt{\epsilon_0/\mu_0} \\ \mathbf{E}_2 &= \hat{\mathbf{i}}\, E_{20} \\ \mathbf{H}_2 & -\hat{\mathbf{j}}\, E_{20}\sqrt{\epsilon_0/\mu_0} \end{aligned} \quad [14]$$

where E_{30} is the complex amplitude of $\mathbf{E}_3$ at 0 and E_{20} is the complex amplitude of $\mathbf{E}_2$ at 0.

Eq. [14] assumes that both $\mathbf{E}_3$ and $\mathbf{E}_2$ are linearly polarized in the same direction. This is justifiable within the narrow common volume illuminated by the two antennas.

Now, over the region occupied by the raindrop, we can express $\mathbf{E}_3,\mathbf{H}_3$ and $\mathbf{E}_2,\mathbf{H}_2$ as approximately plane waves travelling in opposite directions. Thus, taking the point 0 as the origin of the z' axis (see Fig. 2),

$$\begin{aligned} \mathbf{E}_3 &= \hat{\mathbf{i}}\, E_{30} \exp\{-j\, k\, z'\} \\ \mathbf{H}_3 &= \mathbf{j}\, E_{30}\sqrt{\epsilon_0/\mu_0} \exp\{-j\, k\, z'\} \\ \mathbf{E}_2 &= \hat{\mathbf{i}}\, E_{20} \exp\{j\, k\, z'\} \\ \mathbf{H}_2 &= -\hat{\mathbf{j}}\, E_{20}\sqrt{\epsilon_0/\mu_0} \exp\{j\, k\, z'\} \end{aligned} \quad [15]$$

From the above, it can be seen that

$$\begin{aligned} \mathbf{E}_2 &= \left(\frac{E_{20}}{E_{30}^*}\right) \mathbf{E}_3^* \\ \mathbf{H}_2 &= \left(\frac{E_{20}}{E_{30}^*}\right) \mathbf{H}_3^* \end{aligned} \quad [16]$$

where * denotes the complex conjugate. Using Eq. [16], we have

$$\iint_{S_6} (\mathbf{E}_{S_1} \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_{S_1}) \cdot \hat{\mathbf{n}} dS = -\left(\frac{E_{20}}{E_{30}^*}\right) \iint_{S_6} (\mathbf{E}_3^* \times \mathbf{H}_{S_1} + \mathbf{E}_{S_1} \times \mathbf{H}_3^*) \cdot \mathbf{n} dS. \quad [17]$$

It is now recalled that the scattered field $\mathbf{E}_{S_1},\mathbf{H}_{S_1}$ is in fact due to $\mathbf{E}_3,\mathbf{H}_3$ incident on the raindrop. The integral on the right hand side of Eq. [17] can be shown[12] to be related to the forward scattering complex vector amplitude $\mathbf{F}(0)$ of a scatterer by the following (see Appendix 2):

$$\iint_{S_6} (\mathbf{E}_3^* \times \mathbf{H}_{S_1} + \mathbf{E}_{S_1} \times \mathbf{H}_3^*) \cdot \hat{\mathbf{n}} dS = -\frac{4\pi}{jk} \sqrt{\epsilon_0/\mu_0}\, (\mathbf{E}_{30}^* \cdot \mathbf{F}(0)). \quad [18]$$

For any scatterer, we have*

$$\mathbf{F}(0) = \hat{\mathbf{i}}\, E_{30} \frac{S(0)}{jk} \qquad [19]$$

where $S(0)$ is the forward scattering complex scalar amplitude of a scatterer as defined by Van de Hulst (see Appendix 1).

Using Eqs. [17], [18], and [19] and remembering that $\mathbf{E}_{30}^* = \hat{\mathbf{i}}\, E_{30}^*$, we obtain,

$$\iint_{S_6} (\mathbf{E}_{S_1} \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_{S_1}) \cdot \hat{\mathbf{n}} dS = -\frac{4\pi}{k^2} \sqrt{\epsilon_0/\mu_0}\, E_{30}\, E_{20}\, S(0). \qquad [20]$$

Finally, using Eqs. [11], [12], [13], and [20] in Eq. [10], we have

$$-2 \frac{A\, B_s}{Z_0} \iint_{S_2} f^2(x,y)\, dS - \frac{4\pi}{k^2} \sqrt{\epsilon_0/\mu_0}\, E_{30}\, E_{20}\, S(0) = 0,$$

i.e.,

$$B_s = -2 \frac{\pi\, Z_0}{A\, k^2} \sqrt{\epsilon_0/\mu_0} \left(\frac{E_{30}\, E_{20}\, S(0)}{\iint_{S_2} f^2(x,y)\, dS} \right). \qquad [21]$$

2.3 E. M. Wave Propagation In the Presence of Scatterers In An Elemental Volume

From Eq. [15] it is noted that in the region occupied by the raindrop,

$$\mathbf{E}_3 = \hat{\mathbf{i}}\, E_3 = \hat{\mathbf{i}}\, E_{30} \exp\{-j\, k\, z'\}$$
$$\mathbf{E}_2 = \hat{\mathbf{i}}\, E_2 = \hat{\mathbf{i}}\, E_{20} \exp\{j\, k\, z'\} \qquad [22]$$

where E_3 and E_2 are the complex scalar amplitudes of $\mathbf{E}_3$ and $\mathbf{E}_2$, respectively. Hence, over this region,

$$E_{30}\, E_{20} = E_3\, E_2. \qquad [23]$$

The above equation shows that the product E_3E_2 can be assumed constant over the region occupied by the raindrop. In fact it can be assumed that E_3E_2 is constant over an elemental volume dv in space. The wave picked up by the receiving antenna from any rain-

* F(0) and S(0) are dependent on the scatterer shape

drop located within this volume is thus obtained by using Eqs. [23] and [21] as,

$$B_s = -2\,\frac{\pi Z_0}{A\,k^2}\sqrt{\epsilon_0/\mu_0}\left(\frac{E_3E_2\,S(0)}{\iint_{S_2} f^2(x,y)\,dS}\right). \qquad [24]$$

If the elemental volume dv contains similar sized raindrops uniformly distributed in dv and if N is the number of raindrops per unit volume, then by applying Single Scattering theory it follows from Eq. [24] that the wave picked up by the receiving antenna due to scattered radiation from the raindrops in dv is given by

$$B_S^{dv} = -2\,\frac{\pi Z_0}{A\,k^2}\sqrt{\epsilon_0/\mu_0}\left(\frac{E_3E_2\,N\,S(0)}{\iint_{S_2} f^2(x,y)\,dS}\right)dv \qquad [25]$$

2.4 E. M. Wave Propagation in the Presence of a Distribution of Scattering

The analysis is now extended to include a precipitation region containing many raindrops. Consider the arrangement shown in Fig. 3. Let the precipitation be contained between the planes $z = z_1$ and $z = z_2$. Further, let the elemental volume dv be located at plane z.

It is recalled that in Eq. [25], E_3 is taken to be the complex scalar amplitude of $\mathbf{E}_3$ in the elemental volume dv. Further, $\mathbf{E}_3,\mathbf{H}_3$ was tacitly assumed in Sec. 2.2 to be the field due to the transmitting antenna (i.e., antenna 1) in the presence of precipitation. In fact it is this field which induces the scattered field $\mathbf{E}_{S_1},\mathbf{H}_{S_1}$ of each raindrop.

Now, when no precipitation is present, $\mathbf{E}_3,\mathbf{H}_3$ is identical to the field $\mathbf{E}_1,\mathbf{H}_1$ in Sec. 2.1. When precipitation is present, $\mathbf{E}_3$ and $\mathbf{E}_1$ are related via the effective complex refractive index $\tilde{m}$ of the precipitation region (see Appendix 1) as,

$$\mathbf{E}_3 = \mathbf{E}_1 \exp\{-jk\,(\tilde{m} - 1)\,(z - z_1), \qquad [26]$$

where, $z_1 \leqslant z \leqslant z_2$. With reference to Fig. 3, Eq. [26] gives the transmitter field (i.e., the field due to antenna 1 in the presence of precipitation) in the elemental volume dv. Hence, using Eq. [25], we obtain the wave picked up by the receiving antenna due to scattering by raindrops in dv,

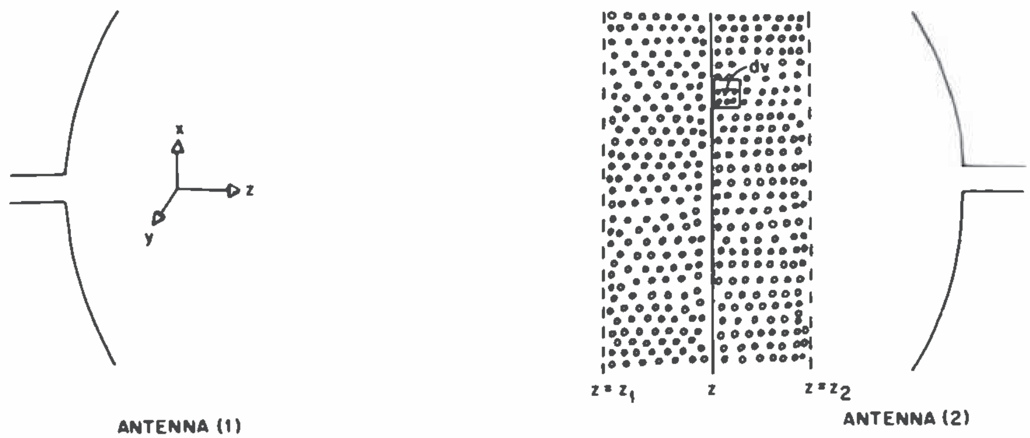

Fig. 3—A distribution of scatterers.

$$B_S^{dv} = -2\,\frac{\pi Z_0}{A\,k^2}\sqrt{\epsilon_0/\mu_0}\left(\frac{E_1\,E_2\,N\,S(0)\exp\{-jk\,(\bar{m}-1)\,(z-z_1)\}}{\iint_{S_2} f^2(x,y)\,dS}\right) \quad [27]$$

where E_1 and E_2 are taken as the complex scalar amplitudes of the field vectors $\mathbf{E}_1$ and $\mathbf{E}_2$, respectively, in dv.

Assuming single scattering and that a sufficient number of drops are present in the elemental volume, the wave picked up by the receiving antenna, due to scattering from all the raindrops in the precipitation region, is obtained by integrating Eq. [27], i.e.,

$$\Sigma B_S^{dv} = -2\,\frac{\pi Z_0}{A\,k^2}\sqrt{\epsilon_0/\mu_0}\left(\frac{\iiint_{vol} E_1\,E_2\,N\,S(0)\exp\{-jk\,(\bar{m}-1)\,(z-z_1)\}}{\iint_{S_2} f^2(x,y)\,dS}\,dv\right) \quad [28]$$

where vol indicates integration over the precipitation volume, i.e., the region between $z = z_1$ and $z = z_2$.

Now, the beamwidth of the two antennas, transmitter and receiver, is very small. Therefore, the volume integral in Eq. [28] may be written

$$\int_{z_1}^{z_2}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_1\,E_2\,dx\,dy\right] N\,S(0)\exp\{-jk\,(\bar{m}-1)\,(z-z_1)\}\,dz \quad [29]$$

and Eq. [28] becomes

$$\Sigma B_S^{dv} = -2 \frac{\pi Z_0}{A k^2} \sqrt{\epsilon_0/\mu_0} \int_{z_1}^{z_2} \left(\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_1 E_2 \, dx \, dy \right] N S(0) \right.$$
$$\left. \exp\{-jk(\tilde{m} - 1)(z - z_1)\} \left[\iint_{S_2} f^2(x,y) \, dS \right]^{-1} dz \right) \qquad [30]$$

At this stage it is useful to draw attention to two important equations. In Sec. 2.1 it was shown that the received wave in the absence of precipitation is given by

$$B_i = \frac{Z_0}{A} \sqrt{\epsilon_0/\mu_0} \left(\frac{\iint_{S_1} E_1 E_2 \, dS}{\iint_{S_2} f^2(x,y) \, dS} \right). \qquad [9]$$

The other important result is Eq. [30]. This gives the received wave due to scattering from all the raindrops in the precipitation region. The total wave received in the presence of precipitation is

$$B_T = B_i + \Sigma B_S^{dv}$$
$$= B_i \left[1 + \Sigma \frac{B_S^{dv}}{B_i} \right]. \qquad [31]$$

Now, from Eqs. [9] and [30], we obtain

$$\Sigma \frac{B_S^{dv}}{B_i} = -2 \frac{\pi}{k^2} \int_{z_1}^{z_2} \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_1 E_2 \, dx \, dy \right] N S(0)$$
$$\exp\{-jk(\tilde{m} - 1)(z - z_1)\} \left[\iint_{S_2} E_1 E_2 \, dS \right]^{-1} dz \qquad [32]$$

Using this in Eq. [31], we have

$$B_T = B_i \left[1 - \frac{2\pi}{k^2} \int_{z_1}^{z_2} \left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_1 E_2 \, dx \, dy \right] N S(0) \right.$$
$$\left. \exp\{-jk(\tilde{m} - 1)(z - z_1)\} \left[\iint_{S_2} E_1 E_2 \, dS \right]^{-1} dz \right] \qquad [33]$$

If we define α and β as the total attenuation in Nepers and phase shift in radians, respectively, due to the precipitation, then

$$B_T = B_i \exp\{-(\alpha + j\beta)\}. \qquad [34]$$

Hence, comparing Eqs. [33] and [34], we obtain

$$\exp\{-(\alpha + j\beta)\} = 1 - \frac{2\pi}{k^2}\int_{z_1}^{z_2}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_1\, E_2\, dx\, dy\right] N\, S(0) \exp\{-jk\,(\tilde{m} - 1)\,(z - z_1)\} \left[\iint_{S_1} E_1\, E_2\, dS\right]^{-1} dz \qquad [35]$$

Now, $\tilde{m}$ and S(0) are related by Eq. [A7] (see Appendix 1). Using Eq. [A7] in Eq. [35] gives

$$\exp\{-(\alpha + j\beta)\} = 1 - \int_{z_1}^{z_2}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_1\, E_2\, dx\, dy\right] jk\,(\tilde{m} - 1) \exp\{-jk\,(\tilde{m} - 1)\,(z - z_1)\} \left[\iint_{S_1} E_1\, E_2\, dS\right]^{-1} dz \qquad [36]$$

In naturally occurring rain, the raindrops will have a drop size distribution, and $\tilde{m}$ will be given by,

$$\tilde{m} = 1 - j\,\frac{2\pi}{k^3}\sum_a N\,(a)\, S(0)_a \qquad [37]$$

where a is the mean drop radius. If the rate of precipitation is uniform throughout, $\tilde{m}$ will be independent of z and Eq. [36] becomes,

$$\exp\{-(\alpha + j\beta)\} = 1 - jk\,(\tilde{m} - 1) \exp\{jk\,(\tilde{m} - 1)\, z_1\} \int_{z_1}^{z_2}\left[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_1\, E_2\, dx\, dy\right] \exp\{-jk\,(\tilde{m} - 1)\, z\} \left[\iint_{S_1} E_1\, E_2\, dS\right]^{-1} dz \qquad [38]$$

The integral $\iint_{S2} E_1\, E_2\, dS$ predicts the receiving power at antenna 1 due to antenna 2 transmitting (see Fig. 1). The integral $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_1\, E_2\, dS$ can be written as $\iint_A E_1\, E_2\, dS$, where A is the beamwidth cross section of the common volume of propagation of the two antennas. Then

$$\iint_A E_1\, E_2\, dS = \iint_{S_1} E_1\, E_2\, dS.$$

Therefore

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} E_1 E_2 \, dS = \iint_{S_1} E_1 E_2 \, dS \qquad [39]$$

for any plane z normal to the direction of propagation. Using Eq. [39] in Eq. [38] we obtain

$$\exp\{-(\alpha + j\beta)\} = 1 - jk(\bar{m} - 1)\int_{z_1}^{z_2} \exp\{-jk(\bar{m} - 1)(z - z_1)\}\, dz$$
$$= \exp\{-jk(\bar{m} - 1)(z_2 - z_1)$$

and

$$\alpha + j\beta = j\,k\,(\bar{m} - 1)(z_2 - z_1). \qquad [40]$$

Hence

$$\alpha = \text{Re}\,[j\,k\,(\bar{m} - 1)(z_2 - z_1)] \text{ Nepers}$$
$$= 8.686\,k\,(z_2 - z_1)\,\text{Im}\,(1 - \bar{m}) \text{ dB} \qquad [41]$$

$$\beta = \text{Im}\,[j\,k\,(\bar{m} - 1)(z_2 - z_1) \text{ Radians}$$
$$= k\,(z_2 - z_1)\,\text{Re}\,(\bar{m} - 1). \text{ Radians} \qquad [42]$$

Eqs. [41] and [42] are the same as Eqs. [A9] and [A10] in Appendix 1 for the case where the scatterers are in the far field of both transmitting and receiving antennas. Thus, whether rain is in the far field or the near field of each antenna, the result is the same total attenuation and total phase shift.

3. Conclusion

In this paper a new theoretical approach to the problem of electromagnetic wave propagation through a medium containing a distribution of scatterers was presented. The new theory includes the effects of scatterers in the near-field regions of the transmitting and receiving antennas, a factor that the currently accepted theory of Van de Hulst[7] does not include. Near-field effects are important especially in the case of satellite microwave communications where the near field of the large ground station antenna includes an appreciable fraction or even the whole of the rain path.

The conclusion from the analysis presented is that scatterers, such as rain in a normal precipitation environment, whether in the near or the far field of the transmitting and receiving antennas, will introduce the same attenuation and phase shift, and therefore the same depolarization.

From the foregoing, it is evident that R. K. Crane's[10] conclusion that the near fields of the antennas must be included in the calculation of the attenuation and phase-shift effects due to precipitation, when such near fields are an appreciable part of the rain path (i.e., in a satellite link), could be in error.

Appendix 1—Wave Propagation in a Medium Containing Scatterers

Let a fixed particle of arbitrary shape and composition be illuminated by a plane scalar wave. The origin of coordinates is chosen somewhere in the particle. The disturbance of the incident wave can be expressed by

$$u_0 = \exp\{-j\,k\,z\} \qquad [A1]$$

The scattered spherical wave in the forward direction in the distant field is then given by,

$$u = \frac{S(0)}{j\,k\,r}\exp\{-j\,k\,r\} \qquad [A2]$$

where r is the distance from the particle to the point of observation and $S(0)$ the forward scattering scalar complex amplitude of a single scatterer. Combining Eqs. [A1] and [A2], we have

$$u = \frac{S(0)}{j\,k\,r}\exp\{-j\,k\,r + j\,k\,z\}\,u_0 \qquad [A3]$$

If the point of observation is (x,y,z), then if x and y are much smaller than z, we obtain

$$r = z + \frac{x^2 + y^2}{2z}$$

Adding the amplitudes of u_0 and u of the incident and scattered waves we obtain,

$$u_0 + u = u_0\,[1 + (S(0)/j\,k\,z)\exp\{-jk\,([x^2 + y^2]/2z)\} \qquad [A4]$$

In the case of a medium containing many scatterers, which are all identical and identically orientated, only particles in the 'active' volume, i.e., the volume of propagation between the transmitting and receiving antenna, which coincides with the few central Freznel zones as seen from the observation point, will influence the forward travelling wave. The total amplitude at the observation point is then given by

$$u = u_0 \left[1 + S(0) \sum_N \frac{1}{j\,k\,z} \exp\left\{ -jk\left(\frac{x^2 + y^2}{2z}\right)\right\}\right]$$

where the summation Σ_N is extended over all the particles in the 'active' volume. If these particles are numerous, the summation sign may be replaced by

$\int N\, dx\, dy\, dz.$

Direct integration gives

$$u = u_0 \left[1 - \frac{2\pi}{k^2} N\, l\, S(0) \right], \qquad \text{[A5]}$$

where l is the length of precipitation in the z-direction. The result may formally be represented as the influence of a complex refractive index of the medium containing scatterers as a whole. If we assume a complex refractive index $\tilde{m}$ for the medium, then, the amplitude of the wave is changed by the medium in the proportion,

$$\exp\{-j\,k\,l\,(\tilde{m} - 1)\} = 1 - j\,k\,l\,(\tilde{m} - 1). \qquad \text{[A6]}$$

Also, from Eqs. [A5] and [A6] we obtain

$$\tilde{m} = 1 - j\frac{2\pi}{k^3} N\, S(0). \qquad \text{[A7]}$$

From Eq. [A6], we have

$$\exp\{-(\alpha + j\beta)\} = \exp\{-j\,k\,l\,(\tilde{m} - 1)\}, \qquad \text{[A8]}$$

where α is the total attenuation in nepers and β is the total phase shift in radians. From Eq. [A8] we obtain

$$\begin{aligned} \alpha &= \mathrm{Re}\,[j\,k\,(\tilde{m} - 1)\,l]\ \text{Nepers} \\ &= 8.686\,k\,l\,\mathrm{Im}\,(1 - \tilde{m})\ \text{dB} \end{aligned} \qquad \text{[A9]}$$

$$\begin{aligned} \beta &= \mathrm{Im}\,[j\,k\,(\tilde{m} - 1)\,l] \\ &= k\,l\,\mathrm{Re}\,(\tilde{m} - 1)\ \text{Radians} \end{aligned} \qquad \text{[A10]}$$

Eqs. [A9] and [A10] are the same as Eqs. [41] and [42] in Sec. 2.4.

Appendix 2—The Forward Scattering Complex Vector Amplitude

Consider

$$\iint_S (\mathbf{E}_i^* \times \mathbf{H}_S + \mathbf{E}_S \times \mathbf{H}_i^*) \cdot \hat{\mathbf{u}}_n\, dS \qquad \text{[A11]}$$

where $\hat{\mathbf{u}}_n$ is the outward normal on the surface S. This is Eq. [21] in Sec. 2.2 where

$$E_i = \mathbf{A} \exp\{-jk\, \mathbf{u} \cdot \mathbf{r}\}$$

$$H_i = \sqrt{\epsilon_0/\mu_0}\, (\mathbf{u} \times \mathbf{A}) \exp\{-jk\, \mathbf{u} \cdot \mathbf{r}\}. \qquad [A12]$$

where $\mathbf{u}$ is the unit vector in the direction of observation. Using Eq. [A12] in [A11], we obtain

$$\iint_S (\mathbf{E}_i^* \times \mathbf{H}_S + \mathbf{E}_S \times \mathbf{H}_i^*) \cdot \hat{\mathbf{u}}_n\, dS = \iint_S (\mathbf{A}^* \times \mathbf{H}_S) \cdot \hat{\mathbf{u}}_n \exp\{-jk\, \mathbf{u} \cdot \mathbf{r}\}\, dS + \sqrt{\epsilon_0/\mu_0} \iint_S [\mathbf{E}_S \times (\mathbf{u} \times \mathbf{A})] \cdot \hat{\mathbf{u}}_n \exp\{-jk\, \mathbf{u} \cdot \mathbf{r}\}\, dS. \qquad [A13]$$

Now

$$\begin{aligned}(\mathbf{A}^* \times \mathbf{H}_S) \cdot \hat{\mathbf{u}}_n &= \hat{\mathbf{u}}_n \cdot [\mathbf{A}^* \times \mathbf{H}_S] \\ &= \mathbf{A}^* \cdot [\mathbf{H}_S + \hat{\mathbf{u}}_n] \\ &= -\mathbf{A}^* \cdot (\hat{\mathbf{u}}_n \times \mathbf{H}_S) \end{aligned} \qquad [A14]$$

and

$$\begin{aligned}[\mathbf{E}_S \times (\mathbf{u} \times \mathbf{A}^*)] \cdot \hat{\mathbf{u}}_n &= \hat{\mathbf{u}}_n \cdot [\mathbf{E}_S \times (\mathbf{u} \times \mathbf{A}^*)] \\ &= (\mathbf{u} \times \mathbf{A}^*) \cdot (\hat{\mathbf{u}}_n \times \mathbf{E}_S) \\ &= (\hat{\mathbf{u}}_n \times \mathbf{E}_S) \cdot (\mathbf{u} \times \mathbf{A}^*) \\ &= \mathbf{A}^* \cdot [(\hat{\mathbf{u}}_n \times \mathbf{E}_S) \times \mathbf{u}] \\ &= -\mathbf{A}^* \cdot [\mathbf{u} \times (\hat{\mathbf{u}}_n \times \mathbf{E}_S)] \end{aligned} \qquad [A15]$$

Using Eqs. [A14] and [A15] in Eq. [A13] and defining for convenience $e^K = e^{jk\mathbf{u} \cdot \mathbf{r}}$, we have

$$\begin{aligned}\iint_S (\mathbf{E}_i^* \times \mathbf{H}_S + \mathbf{E}_S \times \mathbf{H}_i^*) \cdot \hat{\mathbf{u}}_n\, dS &= -\mathbf{A}^* \cdot \iint_S (\hat{\mathbf{u}}_n \times \mathbf{H}_S)\, e^K\, dS \\ &\quad - \sqrt{\epsilon_0/\mu_0}\, \mathbf{A}^* \cdot \iint_S [\mathbf{u} \times \hat{\mathbf{u}}_n \times \mathbf{E}_S]\, e^K\, dS \\ &= -\mathbf{A}^* \cdot \iint_S (\hat{\mathbf{u}}_n \times \mathbf{H}_S)\, e^K\, dS - \sqrt{\epsilon_0/\mu_0}\, \mathbf{A}^* \cdot \left[\mathbf{u} \times \iint_S (\hat{\mathbf{u}}_n \times \mathbf{E}_S)\, e^K\, dS \right] \\ &= \mathbf{A}^* \cdot \left[-\iint_S (\hat{\mathbf{u}}_n \times \mathbf{H}_S)\, e^K\, dS - \sqrt{\epsilon_0/\mu_0} \right.\end{aligned}$$

$$\mathbf{u} \times \iint_S (\hat{\mathbf{u}}_n \times \mathbf{E}_S)\, e^K\, dS \Bigg] . \qquad \text{[A16]}$$

From Eq. (8.105) of Ref. [12], we have for the forward scattering complex vector amplitude $\mathbf{F}_t(\mathbf{u})$

$$\mathbf{F}_t(\mathbf{u}) = \frac{jk}{4\pi} \Bigg[\sqrt{\mu_0/\epsilon_0}\, \mathbf{u} \times \Bigg[\mathbf{u} \times \iint_S (\hat{\mathbf{u}}_n \times \mathbf{H}_S)\, e^K\, dS \Bigg] - \mathbf{u} \times \iint_S (\hat{\mathbf{u}}_n \times \mathbf{E}_S)\, e^K\, dS$$

$$= \frac{jk}{4\pi} \sqrt{\mu_0/\epsilon_0} \Bigg[\mathbf{u} \times \Bigg[\mathbf{u} \times \iint_S (\hat{\mathbf{u}}_n \times \mathbf{H}_S)\, e^K\, dS \Bigg] - \sqrt{\epsilon_0/\mu_0}\, \mathbf{u} \times \iint_S (\hat{\mathbf{u}}_n \times \mathbf{E}_S)\, e^K\, dS \Bigg]$$

$$= \frac{jk}{4\pi} \sqrt{\epsilon_0/\mu_0} \Bigg[\mathbf{u} \Bigg[\mathbf{u} \cdot \iint_S (\hat{\mathbf{u}}_n \times \mathbf{H}_S)\, e^K\, dS \Bigg] - (\mathbf{u} \cdot \mathbf{u}) \iint_S (\hat{\mathbf{u}}_n \times \mathbf{H}_S)\, e^K\, dS - \sqrt{\epsilon_0/\mu_0}\, \mathbf{u} \times \iint_S (\hat{\mathbf{u}}_n \times \mathbf{E}_S)\, e^K\, dS \Bigg] . \qquad \text{[A17]}$$

Then

$$\mathbf{A}^* \cdot \mathbf{F}_t(\mathbf{u}) = \frac{jk}{4\pi} \sqrt{\mu_0/\epsilon_0} \Bigg[\mathbf{A}^* \cdot \mathbf{u} \Bigg[\mathbf{u} \cdot \iint_S (\hat{\mathbf{u}}_n \times \mathbf{H}_S)\, e^K\, dS \Bigg] - \mathbf{A}^* \cdot \iint_S (\hat{\mathbf{u}}_n \times \mathbf{H}_S)\, e^K\, dS - \sqrt{\epsilon_0/\mu_0}\; \mathbf{A}^* \cdot \Bigg[\mathbf{u} \times \iint_S (\hat{\mathbf{u}}_n \times \mathbf{E}_S)\, e^K\, dS \Bigg]\Bigg]$$

$$= \frac{jk}{4\pi} \sqrt{\mu_0/\epsilon_0} \Bigg[-\mathbf{A}^* \cdot \iint_S (\hat{\mathbf{u}}_n \times \mathbf{H}_S)\, e^K\, dS - \sqrt{\epsilon_0/\mu_0}\, \mathbf{A}^* \cdot \Bigg[\mathbf{u} \times \iint_S (\hat{\mathbf{u}}_n \times \mathbf{E}_S)\, e^K\, dS \Bigg]\Bigg]$$

$$= \frac{jk}{4\pi} \sqrt{\mu_0/\epsilon_0}\, \mathbf{A}^* \cdot \left[- \iint_S (\hat{\mathbf{u}}_n \times \mathbf{H}_S)\, e^K\, dS - \sqrt{\epsilon_0/\mu_0}\, \mathbf{u} \times \iint_S (\hat{\mathbf{u}}_n \times \mathbf{E}_S)\, e^K\, dS \right] \quad [A18]$$

Using Eq. [A16] in Eq. [A18], we obtain

$$\mathbf{A}^* \cdot \mathbf{F}_t(\mathbf{u}) = \frac{jk}{4\pi} \sqrt{\mu_0/\epsilon_0} \left[\iint_S (\mathbf{E}_i^* \times \mathbf{H}_S + \mathbf{E}_S \times \mathbf{H}_i) \cdot \hat{\mathbf{u}}_n\, dS \right]$$

Thus

$$\iint_S (\mathbf{E}_i^* \times \mathbf{H}_S + \mathbf{E}_S \times \mathbf{H}_i^*) \cdot \hat{\mathbf{u}}_n\, dS = \frac{4\pi}{jk} \sqrt{\epsilon_0/\mu_0}\, [\mathbf{A}^* \cdot \mathbf{F}_t(\mathbf{u})] \quad [A19]$$

If S is S_6, the surface of the scatterer in Sec. 2.2, then

$$\iint_{S_6} (\mathbf{E}_i^* \times \mathbf{H}_S + \mathbf{E}_S \times \mathbf{H}_i^*) \cdot \hat{\mathbf{n}}\, dS = - \iint_{S_6} (\mathbf{E}_i^* \times \mathbf{H}_S + \mathbf{E}_S \times \mathbf{H}_i^*) \cdot \hat{\mathbf{u}}_n\, dS$$

where $\hat{\mathbf{u}}_n = -\hat{\mathbf{n}}$. Therefore

$$\iint_{S_6} (\mathbf{E}_i^* \times \mathbf{H}_S + \mathbf{E}_S \times \mathbf{H}_i^*) \cdot \hat{\mathbf{n}}\, dS = - \frac{4\pi}{jk} \sqrt{\epsilon_0/\mu_0}\, [\mathbf{A}^* \cdot \mathbf{F}_t(\mathbf{u})] \quad [A20]$$

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