CROSSPOLARIZATION DISCRIMINATION ESTIMATES FOR TERRESTRIAL AND SATELLITE PATHS USING A STATISTICAL RAINDROP CANTING ANGLE MODEL

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Abstract

The statistical model developed by Howard (1,2) is applied to terrestrial and satellite links. Estimates of crosspolarisation discrimination values in both cases are calculated. It is found that for satellite paths crosspolarisation discrimination variations for rain alone are higher than in the terrestrial case.

Introduction

In this paper the statistical model by Howard (1) is used to calculate variations of crosspolarisation discrimination values for terrestrial and satellite links. The statistical model considered here relates turbulence to the reaction of the raindrop canting angle to this turbulence. It is assumed that in general a wind spectrum exists and that within the turbulent boundary layer, the direction of the mean wind is constant (3) for the case of neutral or near neutral atmospheric conditions (i.e., purely mechanical turbulence) a case most appropriate in a precipitation environment (4,5). Using the differential equation describing the horizontal drop movement by canting angle 'Transfer Function' is developed. By applying the horizontal wind velocity spectrum as 'input' and by using Taylor's series (6) the 'response' of the drop (i.e., the mean and variance of its canting angle) is estimated. Application of the model on terrestrial and satellite links shows that large variations of the crosspolarisation discrimination values may exist especially in satellite to earth paths.

Theory

The spectral components for the one dimensional turbulence may be given for any frequency f by,

\[ U(t) = U(f) \cos(\omega t + \alpha(f)) \]  

(1)

where, \( U(f) \), \( \alpha(f) \) are random variables of amplitude and phase and \( U(t) \) represents an ensemble of sample wind velocity functions.

The differential equations for the horizontal drop movement is given by (7),
where all symbols are defined in Ref. (7).

Applying Eq. (1) in Eq. (2) we obtain,

\[ V(t) = V(f) \cos \omega t + \beta(f) \]  

(3)

In this case \( V(f) \) and \( \beta(f) \) are random variables of amplitude and phase and \( V(t) \) represents an ensemble of sample raindrop velocity functions.

The canting angle of a raindrop due to wind variation is given in (7) as,

\[ \tan \theta = \frac{U-V}{W} \]  

(4)

Using Eqs. (1) and (3) in Eq. (4) we have,

\[ \tan \theta = \frac{U(f)}{W} \frac{1}{1 + (g/wV)^2} \cos(\omega t + \arctan \left( \frac{g}{wV} \right)) \]  

(5)

In Eq. (5) \( \frac{1}{V(W^2 + (g/wV)^2)} \) is the amplitude of a 'Transfer Function' relating \( \tan \theta \) to a sinusoidal wind variation. The phase delay introduced by the transfer function is given by \( \arctan \left( \frac{g}{wV} \right) \).

The energy spectrum of \( \tan \theta \) can be obtained by,

\[ \sigma^2 = \int 1 \left( 1 + (g/wV)^2 \right) S(f) df \]  

(6)

where, \( S(f) \) is the one dimensional energy spectrum of turbulence (8).

The mean value of \( \tan \theta \) is given in (9) as,

\[ \tan \theta_0 = \frac{U_0}{WV} \exp \left( gh/WV^2 \right) E_1 \left( gh/WV^2 \right) \]  

(7)

with all the symbols defined in Ref. (9).

Using Eqs. (6) and (7) and employing the Taylor's series expansion (1,6) the following estimates for the mean and variance values of the canting angle \( \theta \) are obtained,

\[ E(\theta) = \arctan \left( \tan \theta_0 \right) - \tan \theta_0 \cos^2 \theta_0 \frac{\sigma^2}{\tan \theta} \]  

(8)

\[ \sigma^2 = \cos^2 \theta_0 \frac{\sigma^2}{\tan \theta} + \tan^2 \theta_0 \cos^2 \theta_0 \frac{\sigma^2}{\tan \theta} \]  

(9)

Eqs. (8) and (9) may be used to estimate crosspolarisation discrimination variations in the presence of rain.

The propagation of microwaves in the presence of precipitation may be described by the equation,

\[ \begin{pmatrix} \text{Eh} \\ \text{Ev} \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} \text{Eh} \\ \text{Ev} \end{pmatrix} \]  

(10)

where all the symbols are defined in Ref. (7).

Using the notation of Eq. (10) the crosspolarisation discrimination values for vertical and horizontal polarisations are given by,

\[ \text{XPD}_V = 20 \log_{10} \left| \frac{T_{22}}{T_{12}} \right| \]  

(11)

\[ \text{XPD}_H = 20 \log_{10} \left| \frac{T_{11}}{T_{21}} \right| \]  

(12)

Computations

(i) Application of the model to a terrestrial link

The terrestrial link considered had a path length of 3km and the transmitting and receiving antennas were assumed to be at a height of 10 m. A signal frequency of 11 GHz, a mean horizontal wind velocity of 15 m/s and four precipitation rates, 25, 50, 100 and 150 mm/hr were used. The mean friction velocity \( U_0 \) was assumed to be equal to 1.3 m/s a value associated with a terrain of large open fields (200 - 500 m). The path length was separated into 50 m long layers. The horizontal wind velocities at any two points within each layer were taken as equal, with wind velocities between layers considered completely uncorrelated. The necessary length of the layers was calculated by employing the spatial correlation function of velocity (see Ref. (8)).

Crosspolarisation discrimination calculations were made by using Eq. (10) for each layer and multiplying the layer matrices to evaluate the result. A Gaussian canting angle distribution for each dropsize was assumed. The results are plotted in Figure 1.

(ii) Application of the model to a satellite link

The elevation angle of the satellite path was taken to be equal to 20 degrees. All other data and parameters were the same as in the terrestrial case. The satellite path length was again separated into 50 m long layers since it may be shown that the spatial correlation function varies slowly with height. Care was taken to include the past history of each dropsize. Thus at the
highest (or starting) layer the response of all drops to wind fluctuations is fully correlated, whereas near the ground due to the fact that each drop size falls with different vertical velocity the response of drops in each size is uncorrelated.

Crosspolarisation discrimination estimates are plotted in Figure 1.

Conclusion

In this paper the statistical model by Howard is used to estimate crosspolarisation discrimination values for satellite and terrestrial paths. It is found that for satellite paths there are higher crosspolarisation discrimination variations compared to the terrestrial case.

![Crosspolarisation Discrimination vs Fade](image)

Figure 1: Vertical crosspolarisation discrimination vs. fade. a, c: maximum and minimum bounds, b: average values. —— terrestrial case, —— satellite case.

References

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Frequency Dependence of Microwave Depolarization Versus Rain Attenuation

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It is now well known that the microwave rain-induced cross polarization amplitude is proportional to frequency for a given amount of rain. However, there still exists lingering controversy about the frequency dependence of cross polarization for a given rain attenuation. Previous calculations using oblate spheroidal raindrops, as well as measured data, showed that rain-induced cross polarization amplitude for given rain attenuation is inversely proportional to frequency from 10 to 30 GHz. On the other hand the inverse three-halves power frequency dependence of the CCIR approximate formula is still being claimed by the argument that realistic raindrop shapes, such as those determined by Pruppacher and Pitter, give different frequency dependence from that of oblate raindrops. The purpose of this paper is to resolve the above question on frequency dependence.

Except for extremely low rain rates (≤2.5 mm/hr), analysis of the calculated data for Pruppacher-Pitter raindrops has also shown the rain-induced cross polarization with given attenuation to be inversely proportional to frequency. The results generally agree with those of calculations using oblate spheroidal raindrops which also showed deviations toward 10 GHz at very light rain rate. Deviations at very light rain rate are not important for communication system engineers who are mainly concerned with moderate to heavy rain rates. Possible explanation was also explored for the stray path in approximations leading to the claim of inverse three-halves power frequency dependence. Revision for the frequency dependence of the approximate CCIR formula is indeed supported by overwhelming theoretical and experimental evidences.