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Crosspolarization of Microwaves due to Rain on a Satellite to Earth Path

J. HOWARD AND N. A. MATHEWS

***Abstract*—The effect of rain on the crosspolarization of microwaves on a satellite to earth path has been investigated using a profile formula given by Smith [1]. It is shown that crosspolarization due to rain is less on a satellite to earth path than on an earth to earth one for the same path length.**

At frequencies above 10 GHz, crosspolarization effects due to precipitation in a radio link limit frequency reuse in orthogonal polarizations. It is known [2]–[4] that on a satellite to earth path "anomalous depolarization," that is, high levels of crosspolarization, occurring together with negligible attenuation can exist. It is suggested [2]–[4] that ice crystals are the hydrometeors that cause this anomaly.

An accurate prediction of the effect of rain on a satellite to earth path will help to establish the true depolarization effect that other hydrometeors might have on a propagating wave. It is pointed out that the constant canting-angle model used for earth to earth paths cannot be used for a satellite to earth one. This is because raindrop canting angles are a function of the wind gradient which varies with height [5].

In this communication we apply Brussaard's [5] meteorological model for rain on a satellite to earth path using a more general wind-profile formula to solve the differential equation controlling the horizontal raindrop movement. A raindrop falling through the lower region of the atmosphere (0–2 km) will be constantly decelerated due to the fact that wind speed decelerates with decreasing height. The result of this will be that the raindrop will show a canted orientation (see Fig. 1).

If θ is the canting angle of the raindrop, then

$$\tan\theta = \frac{m(dV_h/dt)}{mg} = \frac{U - V_h}{V_v}, \tag{1}$$

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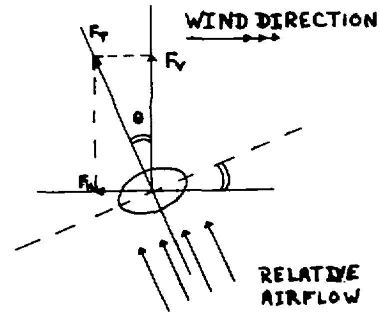

Fig. 1. Raindrop canting. F_T is total drag force on drop, F_H is horizontal component, and F_V is vertical component.

where V_h is the horizontal drop speed, V_v is the vertical drop speed (assumed to be constant and equal to the terminal speed in stagnant air), U is the wind speed at the position of the drop, and g is the gravitational constant.

The differential equation controlling the horizontal movement of the drop is given by

$$\frac{dV_h(t)}{dt} + \frac{gV_h(t)}{V_v} - \frac{gU(t)}{V_v} = 0. \tag{2}$$

The general solution of the above linear differential equation of the first order is

$$V_h(t) = C_1 e^{-(g/V_v)t} + \frac{g}{V_v} e^{-(g/V_v)t} \int_0^t U(\tau) e^{(g/V_v)\tau} d\tau. \tag{3}$$

The vertical profile of the wind speed may be determined by using the logarithmic formula

$$U = \frac{U^*}{K} \ln\left(\frac{Z}{Z_0}\right), \tag{4}$$

where U^* is the surface friction velocity, K is Von Karman's constant ($K \approx 0.4$), and Z_0 is the surface roughness (depending on the type of terrain). It must be noted that the formula applies only down to a height of a multiple of Z_0 ($0.05 < Z_0 < 0.8$ m for flat countryside). Thus the situation where $Z = Z_0$ and the wind speed is zero does not arise.

If there is no updraft, the height of the drop is given by

$$h(t) = h(0) - V_v t. \tag{5}$$

Using (4) and (5) in (2) and imposing the boundary condition

$$\lim_{t \to -\infty} V_h(t) = U(t),$$

(1) becomes

$$\tan\theta = \frac{U^*}{kV_v} e^{(g/V_v{}^2)h} E_1\left(\frac{g}{V_v}h\right), \tag{6}$$

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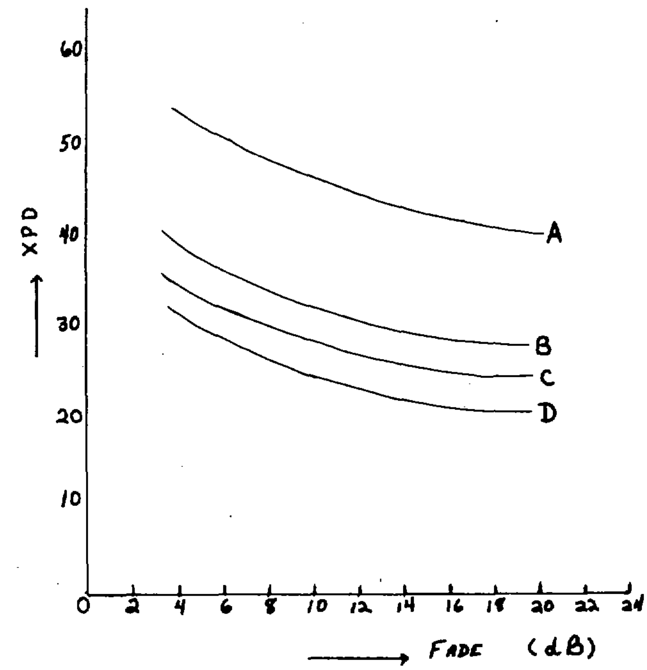

Fig. 2. 11.0 GHz vertical crosspolarization discrimination against fade. *A:* Satellite link. *B:* Ground to ground link—height of antenna 40 m. *C:* Ground to ground link—height of antenna 20 m. *D:* Ground to ground link—height of antenna 10 m. Path length—2.92 km.

where

$$E_1\left(\frac{g}{V_v^2}h\right) = \int_{g/V_v^2 h}^{\infty} \frac{e^{-(g/V_v^2)y}}{g/V_v^2 y}\, d\left(\frac{g}{V_v^2}y\right)$$

is the exponential integral.

Using Brussaard's meteorological rain model [5] the effect of the rain-filled medium on wave propagation can be defined in matrix notation:

$$\begin{bmatrix} E_h \\ E_v \end{bmatrix}_z = \begin{bmatrix} T_{11} & T_{12} \\ T_{12} & T_{22} \end{bmatrix} \begin{bmatrix} E_h \\ E_v \end{bmatrix}_{z=0},$$

where T_{11}, T_{22}, and T_{12} are given by [5, eq. (13)–(15)]. Computations of crosspolarization discriminations were performed at the frequency of 11.0 GHz for precipitations rates of 50, 100, and 150 mm/hr. The wind speed was considered to be 10 m/s at the 10-m height. Z_0 was taken to being equal to 0.25 m. Rainfall was assumed to have been initiated at a height of 1 km. The elevation angle of the earth to satellite path was taken to be equal to 20°.

Fig. 2 shows a comparison between vertical crosspolarization discrimination for a satellite to earth path and for an earth to earth one against fade. From the graph it can be concluded that crosspolarization discrimination due to rain is higher on a satellite to earth path than on an earth to earth one for the same path length.

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Effects of Sizes of Nonspherical Raindrops on Crosspolarization of Transmittted Microwave Signals

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Abstract—**The effect of raindrop shape on the crosspolarization of microwaves has been investigated using a modification of Brussard's meteorological rain model. It is shown that in a rainfall situation the drops that mainly affect the crosspolarization signal level are the small- to medium-size ones.**

Current interest in microwave propagation studies through rain has been prompted by proposals for terrestrial and satellite communication systems operating above 10 GHz. It is well-known that at these frequencies the presence of rain in the transmission medium causes attenuation and depolarization of the transmitted radiation.

It has been pointed out by Taur [1] that the larger drops have a significant effect on the depolarization of microwaves at high rain rates due to the fact that there are more larger drops at high rain rates and that the oblateness of the drop increases with drop size. This has also been suggested by Brussaard [2]. In his paper, Brussaard offers a physical explanation of raindrop canting and shows that the larger drops have greater canting angles than the smaller ones. He points out that the "effective canting angle," i.e., the angle over which the medium has to be rotated around the propagation direction in order to produce cross-polar terms equal to zero, is, for a given precipitation rate, almost equal to the physical canting angle of the largest drops. He therefore concludes that with increasing rain rate the larger drops will contribute more to the depolarization of microwaves. Our calculations, however, indicate that it is not the large drops but the small- to medium-size ones (up to 2.0 mm in radius) that contribute most to depolarization.

A modification of Brussaard's meteorological rain model is used in this communication to demonstrate the above effect. In this model the drops are assumed to have a size distribution according to Laws and Parsons [3] and fall with terminal velocities given by Medhurst [4]. The physical canting angle is

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