where

\[ E_1 \left( \frac{g}{V_0^2} \right) = \int_{g/V_0^2 h}^{\infty} e^{-\frac{g}{V_0^2 y}} \frac{d}{g/V_0^2 y} \]

is the exponential integral.

Using Brussaard's meteorological rain model [5] the effect of the rain-filled medium on wave propagation can be defined in matrix notation:

\[
\begin{bmatrix}
E_h \\
E_v
\end{bmatrix} =
\begin{bmatrix}
T_{11} & T_{12} \\
T_{21} & T_{22}
\end{bmatrix}
\begin{bmatrix}
E_h \\
E_v
\end{bmatrix}
\]

where \( T_{11}, T_{22}, \) and \( T_{12} \) are given by [5, eq. (13)-(15)]. Computations of crosspolarization discriminations were performed at the frequency of 11.0 GHz for precipitation rates of 50, 100, and 150 mm/hr. The wind speed was considered to be 10 m/s at the 10-m height. \( Z_0 \) was taken to be equal to 0.25 m. Rainfall was assumed to have been initiated at a height of 1 km. The elevation angle of the earth to satellite path was taken to be equal to 20°.

Fig. 2 shows a comparison between vertical crosspolarization discrimination for a satellite to earth path and for an earth to earth one against fade. From the graph it can be concluded that crosspolarization discrimination due to rain is higher on a satellite to earth path than on an earth to earth one for the same path length.

References


Effects of Sizes of Nonspherical Raindrops on Crosspolarization of Transmitted Microwave Signals

J. HOWARD AND N. A. MATHEWS

Abstract—The effect of raindrop shape on the crosspolarization of microwaves has been investigated using a modification of Brussaard's meteorological rain model. It is shown that in a rainfall situation the drops that mainly affect the crosspolarization signal level are the small- to medium-size ones.

Current interest in microwave propagation studies through rain has been prompted by proposals for terrestrial and satellite communication systems operating above 10 GHz. It is well-known that at these frequencies the presence of rain in the transmission medium causes attenuation and depolarization of the transmitted radiation.

It has been pointed out by Taur [1] that the larger drops have a significant effect on the depolarization of microwaves at high rain rates due to the fact that there are more larger drops at high rain rates and that the oblateness of the drop increases with drop size. This has also been suggested by Brussaard [2]. In his paper, Brussaard offers a physical explanation of raindrop canting and shows that the larger drops have greater canting angles than the smaller ones. He points out that the "effective canting angle," i.e., the angle over which the medium has to be rotated around the propagation direction in order to produce cross-polar terms equal to zero, is, for a given precipitation rate, almost equal to the physical canting angle on the crmpolarization of microwaves. Our calculations, however, indicate that it is not the large drops but the small- to medium-size ones (up to 2.0 mm in radius) that contribute most to depolarization.

A modification of Brussaard's meteorological rain model is used in this communication to demonstrate the above effect. In this model the drops are assumed to have a size distribution according to Laws and Parsons [3] and fall with terminal velocities given by Medhurst [4]. The physical canting angle is

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IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. AP-27, NO. 6, NOVEMBER 1979 891
size-dependent and influenced by the vertical wind gradient in the manner determined by Brussaard [2]. The shape of the drops are taken to be spheroidal up to a certain drop size, but beyond this the spherical drop shape is assumed. Calculations of the crosspolarization signal level are repeated with this model, starting with only those drops in the smallest size range being spheroidal until finally drops in all size ranges are spheroidal. The latter situation corresponds to the model used by Brussaard [2].

For brevity, no symbols are defined here and the reader is referred to the original paper by Brussaard. Following Brussaard, the effect of the rain-filled medium on wave propagation can be defined in matrix notation

\[
\begin{bmatrix}
E_h \\
E_v
\end{bmatrix}
= 
\begin{bmatrix}
T_{11} & T_{12} \\
T_{12} & T_{22}
\end{bmatrix}
\begin{bmatrix}
E_h \\
E_v
\end{bmatrix}
\bigg|_{z=0}
\]

where \(T_{11}, T_{22}, \text{ and } T_{12}\) are given by [2, eq. (13)-(15)]. Our interest here is \(T_{12}\), since this determines the level of the received crosspolarized signal. This matrix element is expressed in terms of the quantities \(G_{hh}, G_{vv}, \text{ and } G_{hv}\), and for the rain model adopted here they are given by the following equations:

\[
G_{hh} = \sum_{i=1}^{K} (S_{II}^i \cos^2 \theta i + S_{I}^i \sin^2 \theta i) Ni + \sum_{i=K+1}^{M} S_0^i Ni
\]

\[
G_{vv} = \sum_{i=1}^{K} (S_{II}^i \sin^2 \theta i + S_{I}^i \cos^2 \theta i) Ni + \sum_{i=K+1}^{M} S_0^i Ni
\]

\[
G_{hv} = \sum_{i=1}^{K} (S_{II}^i - S_{I}^i) \left(\frac{\sin 2\theta i}{2}\right) Ni + \sum_{i=K+1}^{M} S_0^i Ni
\]

where \(S_{II}^i\) and \(S_{I}^i\) are the forward-scattering complex scalar amplitudes corresponding to a spheroidal drop in the \(i\)th size range, and \(S_0^i\) is the scattering amplitude of a spherical drop. \(N_i\) is the number of drops in the \(i\)th size range. Note that drops up to the \(K\)th size range are spheroidal. The situation when all drops are spheroidal occurs when \(K = M\).

Computations of \(T_{12}\) were carried out at 11 GHz for a 1-km path and for rain rates of 50, 100, and 150 mm/hr. Data for \(S_{II}^i\) and \(S_{I}^i\) were taken from Morrison and Cross [5], whereas data for \(S_0^i\) were based on Mie's theory. Canting angles for the different drop sizes were obtained via [2, eq. (8)] assuming \(m = 0.2\) and \(U_{10} = 15\) m/s. The wind direction was taken to be perpendicular to the propagation direction.

Fig. 1 shows plots of the crosspolarized signal level, i.e., \(|T_{12}|\), as a function of the drop size beyond which the drop shape is assumed spherical. It is evident that for the rain rates considered \(|T_{12}|\) approaches its final value to within 2 dB when all drops up to about 2.0 mm in radius are spheroidal. The contribution to the crosspolarized signal of larger drops is therefore minimal.

The above calculations were also performed at 19.3 GHz and 30 GHz. Data for \(S_{II}^i\) and \(S_{I}^i\) at 19.3 GHz were taken from Oguchi [6], whereas those for 30 GHz were taken from Morrison and Cross [5]. No graphs are included here, but similar conclusions regarding the minimal contribution of the large drops to the crosspolarized signal level are still valid.

In conclusion, it can be stated that despite the fact that the large drops have higher forward-scattering amplitudes and canting angles than the small- and medium-size ones, their contribution to the crosspolarized signal can be neglected for all practical purposes. The reason why this is so is due to a significantly greater number of small- and medium-size drops than large drops even for high rain rates.
IEEE TRANSACTIONS ON ANTENNAS AND PROPAGATION, VOL. AP-27, NO. 6, NOVEMBER 1979

REFERENCES


Measurement of Relative Propagation Delay Between C- and K-Band Satellite Loops

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Abstract—Variations of relative propagation delay between C- and K-band satellite loops were measured using a communication satellite. This measurement employed a frequency modulation (FM) and a time division multiple access (TDMA) method. The maximum observed delay change was about 6 ns during the ten-day experimental period. The experimental results show that the delay variations are caused mainly by variations of the total electron content in the ionosphere, and also, a small change of about 0.2 ns due to the spin motion of the satellite is found.

I. INTRODUCTION

New satellite communication systems will be expected, in which two or more frequency bands are used, and switching or synchronization among them is necessary. For realization of these kinds of communication systems, it is necessary to understand relative propagation delay and its variation among the satellite loops of different frequency bands.

The refractive index of the ionosphere is a function of radio wave frequency. Considering this characteristic, variation of relative propagation delay due to the ionosphere was measured by using actual communication signals via the communication satellite (CS) which was launched in December 1977. Obtained experimental results were compared with the total electron content which was measured with the ETS-II. (The CS and the ETS-II are Japanese geostationary satellites.)

In this communication relative propagation delay in the ionosphere is discussed, and two methods of the measurement will be introduced. They are a frequency modulation (FM) method and a time division multiple access (TDMA) method, in addition to the use of the coherency of modulation signals. The obtained experimental results using the CS show that the main cause of the relative propagation delay must be due to the ionosphere, and close agreement between the measured values using the CS and the ETS-II was obtained.

II. DELAY IN THE IONOSPHERE

When there are both C- and K-band transponders on board a satellite such as the CS, the motion of the satellite does not affect the relative propagation delay because the delay changes of both the links are the same. Therefore, it is considered that the factors causing the relative delay variations are a) variations of the total electron content along a radio wave path in the ionosphere, b) atmospheric irregularities, c) rainfall, and d) mechanical changes in a satellite and an earth station such as spin motions and vibrations. In this section the ionospheric effect is discussed since it is considered to have the largest effect.

Considering propagation through the ionosphere, denote a modulated radio wave \( a(t) \) as

\[
a(t) = \rho(t) \cos \left[ \omega_c t + \theta(t) \right]
\]

where \( \rho(t) \) and \( \theta(t) \) are amplitude- and phase-varying with a modulation signal, and \( \omega_c \) is the angular frequency of the carrier signal. Let \( a_r(t) \) be a signal which has passed through the ionosphere and \( \Lambda(\omega) \) the Fourier transform of \( a(t) \), then \( a_r(t) \) is expressed as

\[
a_r(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Lambda(\omega) e^{j(\omega t - \omega_c t)} e^{-j \alpha N_0} d\omega,
\]

where \( C \) is the light velocity, \( r \) is the path length, and \( N_0 \) is the refractive index of the ionosphere. As shown in (2), \( a_r(t) \) suffers delay depending on its carrier frequency. When the carrier frequency is sufficiently high, such as that of the C or K band, \( N_0 \) is represented as

\[
n = 1 - \frac{\alpha N_0}{\omega_c^2}, \quad \alpha = 162\pi^2 \text{[rad}^2 \cdot \text{m}^3/\text{sec}^2]\]

where \( N_0 \) is the electron density in the ionosphere. Then, using the fact that the carrier frequencies are sufficiently higher than the modulating signal frequency, \( a_r(t) \) is given by the following equation:

\[
a_r(t) = \rho \left( t - \frac{r}{C} - \frac{\alpha N_0}{C \omega_c^2} \right) \cos \left( \omega_c \left( t - \frac{r}{C} + \frac{\alpha N_T}{C \omega_c^2} \right) + \theta \left( t - \frac{r}{C} + \frac{\alpha N_T}{C \omega_c^2} \right) \right)
\]

where \( N_T \) is the total electron content along the radio wave path and is given as

\[
N_T = \int_0^r N \, dr.
\]

Equation (4) shows that the amplitude \( (\rho(t)) \) and phase \( (\theta(t)) \) have delays depending on the carrier frequency. Then let \( \Delta N_T \) be the change of the total electron content and \( \omega_c \),